

Score lists in multipartite hypertournaments

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Abstract. Given non-negative integers n_i and α_i with $0 \leq \alpha_i \leq n_i$ ($i = 1, 2, \dots, k$), an $[\alpha_1, \alpha_2, \dots, \alpha_k]$ - k -partite hypertournament on $\sum_1^k n_i$ vertices is a $(k+1)$ -tuple $(U_1, U_2, \dots, U_k, E)$, where U_i are k vertex sets with $|U_i| = n_i$, and E is a set of $\sum_1^k \alpha_i$ -tuples of vertices, called arcs, with exactly α_i vertices from U_i , such that any $\sum_1^k \alpha_i$ subset $\cup_1^k U'_i$ of $\cup_1^k U_i$, E contains exactly one of the $(\sum_1^k \alpha_i)!$ $\sum_1^k \alpha_i$ -tuples whose entries belong to $\cup_1^k U'_i$. We obtain necessary and sufficient conditions for k lists of non-negative integers in non-decreasing order to be the losing score lists and to be the score lists of some k -partite hypertournament.

1 Introduction

Hypergraphs are generalizations of graphs [1]. While edges of a graph are pairs of vertices of the graph, edges of a hypergraph are subsets of the vertex set,

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consisting of at least two vertices. An edge consisting of k vertices is called a k -edge. A k -hypergraph is a hypergraph all of whose edges are k -edges. A k -hypertournament is a complete k -hypergraph with each k -edge endowed with an orientation, that is, a linear arrangement of the vertices contained in the hyperedge. Instead of scores of vertices in a tournament, Zhou et al. [13] considered scores and losing scores of vertices in a k -hypertournament, and derived a result analogous to Landau's theorem [6]. The score $s(v_i)$ or s_i of a vertex v_i is the number of arcs containing v_i and in which v_i is not the last element, and the losing score $r(v_i)$ or r_i of a vertex v_i is the number of arcs containing v_i and in which v_i is the last element. The score sequence (losing score sequence) is formed by listing the scores (losing scores) in non-decreasing order.

The following characterizations of score sequences and losing score sequences in k -hypertournaments can be found in G. Zhou et al. [12].

Theorem 1 *Given two positive integers n and k with $n \geq k > 1$, a non-decreasing sequence $R = [r_1, r_2, \dots, r_n]$ of non-negative integers is a losing score sequence of some k -hypertournament if and only if for each j ,*

$$\sum_{i=1}^j r_i \geq \binom{j}{k},$$

with equality when $j = n$.

Theorem 2 *Given positive integers n and k with $n \geq k > 1$, a non-decreasing sequence $S = [s_1, s_2, \dots, s_n]$ of non-negative integers is a score sequence of some k -hypertournament if and only if for each j ,*

$$\sum_{i=1}^j s_i \geq j \binom{n-1}{k-1} + \binom{n-j}{k} - \binom{n}{k},$$

with equality when $j = n$.

Some recent work on the reconstruction of tournaments can be found in the papers due to A. Iványi [3, 4]. Some more results on k -hypertournaments can be found in [2, 5, 9, 10, 11, 13]. The analogous results of Theorem 1 and Theorem 2 for $[h, k]$ -bipartite hypertournaments can be found in [7] and for $[\alpha, \beta, \gamma]$ -tripartite hypertournaments in [8].

Throughout this paper i takes values from 1 to k and j_i takes values from 1 to n_i , unless otherwise stated.

A k -partite hypergraph is a generalization of k -partite graph. Given non-negative integers n_i and α_i , ($i = 1, 2, \dots, k$) with $n_i \geq \alpha_i \geq 0$ for each i , an $[\alpha_1, \alpha_2, \dots, \alpha_k]$ - k -partite hypertournament (or briefly k -partite hypertournament) M of order $\sum_1^k n_i$ consists of k vertex sets U_i with $|U_i| = n_i$ for each i , ($1 \leq i \leq k$) together with an arc set E , a set of $\sum_1^k \alpha_i$ -tuples of vertices, with exactly α_i vertices from U_i , called arcs such that any $\sum_1^k \alpha_i$ subset $\cup_1^k U'_i$ of $\cup_1^k U_i$, E contains exactly one of the $\left(\sum_1^k \alpha_i\right) \sum_1^k \alpha_i$ -tuples whose α_i entries belong to U'_i .

Let $e = (u_{11}, u_{12}, \dots, u_{1\alpha_1}, u_{21}, u_{22}, \dots, u_{2\alpha_2}, \dots, u_{k1}, u_{k2}, \dots, u_{k\alpha_k})$, with $u_{ij_i} \in U_i$ for each i , ($1 \leq i \leq k, 1 \leq j_i \leq \alpha_i$), be an arc in M and let $h < t$, we let $e(u_{1h}, u_{1t})$ denote to be the new arc obtained from e by interchanging u_{1h} and u_{1t} in e . An arc containing α_i vertices from U_i for each i , ($1 \leq i \leq k$) is called an $(\alpha_1, \alpha_2, \dots, \alpha_k)$ -arc.

For a given vertex $u_{ij_i} \in U_i$ for each i , $1 \leq i \leq k$ and $1 \leq j_i \leq \alpha_i$, the score $d_M^+(u_{ij_i})$ (or simply $d^+(u_{ij_i})$) is the number of $\sum_1^k \alpha_i$ -arcs containing u_{ij_i} and in which u_{ij_i} is not the last element. The losing score $d_M^-(u_{ij_i})$ (or simply $d^-(u_{ij_i})$) is the number of $\sum_1^k \alpha_i$ -arcs containing u_{ij_i} and in which u_{ij_i} is the last element. By arranging the losing scores of each vertex set U_i separately in non-decreasing order, we get k lists called losing score lists of M and these are denoted by $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$ for each i , ($1 \leq i \leq k$). Similarly, by arranging the score lists of each vertex set U_i separately in non-decreasing order, we get k lists called score lists of M which are denoted as $S_i = [s_{ij_i}]_{j_i=1}^{n_i}$ for each i ($1 \leq i \leq k$).

2 Main results

The following two theorems are the main results.

Theorem 3 *Given k non-negative integers n_i and k non-negative integers α_i with $1 \leq \alpha_i \leq n_i$ for each i ($1 \leq i \leq k$), the k non-decreasing lists $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$ of non-negative integers are the losing score lists of a k -partite hypertournament if and only if for each p_i ($1 \leq i \leq k$) with $p_i \leq n_i$,*

$$\sum_{i=1}^k \sum_{j_i=1}^{p_i} r_{ij_i} \geq \prod_{i=1}^k \binom{p_i}{\alpha_i}, \quad (1)$$

with equality when $p_i = n_i$ for each i ($1 \leq i \leq k$).

Theorem 4 *Given k non-negative integers n_i and k non-negative integers α_i with $0 \leq \alpha_i \leq n_i$ for each i ($1 \leq i \leq k$), the k non-decreasing lists $S_i = [s_{ij_i}]_{j_i=1}^{n_i}$ of non-negative integers are the score lists of a k -partite hypertournament if and only if for each p_i , ($1 \leq i \leq k$) with $p_i \leq n_i$*

$$\sum_{i=1}^k \sum_{j_i=1}^{p_i} s_{ij_i} \geq \left(\sum_{i=1}^k \frac{\alpha_i p_i}{n_i} \right) \left(\prod_{i=1}^k \binom{n_i}{\alpha_i} \right) + \prod_{i=1}^k \binom{n_i - p_i}{\alpha_i} - \prod_{i=1}^k \binom{n_i}{\alpha_i}, \quad (2)$$

with equality when $p_i = n_i$ for each i ($1 \leq i \leq k$).

We note that in a k -partite hypertournament M , there are exactly $\prod_{i=1}^k \binom{n_i}{\alpha_i}$ arcs and in each arc only one vertex is at the last entry. Therefore,

$$\sum_{i=1}^k \sum_{j_i=1}^{n_i} d_M^-(u_{ij_i}) = \prod_{i=1}^k \binom{n_i}{\alpha_i}.$$

In order to prove the above two theorems, we need the following Lemmas.

Lemma 5 *If M is a k -partite hypertournament of order $\sum_1^k n_i$ with score lists $S_i = [s_{ij_i}]_{j_i=1}^{n_i}$ for each i ($1 \leq i \leq k$), then*

$$\sum_{i=1}^k \sum_{j_i=1}^{n_i} s_{ij_i} = \left[\left(\sum_{i=1}^k \alpha_i \right) - 1 \right] \prod_{i=1}^k \binom{n_i}{\alpha_i}.$$

Proof. We have $n_i \geq \alpha_i$ for each i ($1 \leq i \leq k$). If r_{ij_i} is the losing score of $u_{ij_i} \in U_i$, then

$$\sum_{i=1}^k \sum_{j_i=1}^{n_i} r_{ij_i} = \prod_{i=1}^k \binom{n_i}{\alpha_i}.$$

The number of $[\alpha_i]_1^k$ arcs containing $u_{ij_i} \in U_i$ for each i , ($1 \leq i \leq k$), and $1 \leq j_i \leq n_i$ is

$$\frac{\alpha_i}{n_i} \prod_{t=1}^k \binom{n_t}{\alpha_t}.$$

Thus,

$$\begin{aligned}
 \sum_{i=1}^k \sum_{j_i=1}^{n_i} s_{ij_i} &= \sum_{i=1}^k \sum_{j_i=1}^{n_i} \left(\frac{\alpha_i}{n_i} \right) \prod_1^k \binom{n_t}{\alpha_t} - \binom{n_i}{\alpha_i} \\
 &= \left(\sum_{i=1}^k \alpha_i \right) \prod_1^k \binom{n_t}{\alpha_t} - \prod_1^k \binom{n_i}{\alpha_i} \\
 &= \left[\left(\sum_{i=1}^k \alpha_i \right) - 1 \right] \prod_1^k \binom{n_i}{\alpha_i}.
 \end{aligned}$$

□

Lemma 6 *If $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$ ($1 \leq i \leq k$) are k losing score lists of a k -partite hypertournament M , then there exists some h with $r_{1h} < \frac{\alpha_1}{n_1} \prod_1^k \binom{n_p}{\alpha_p}$ so that $R'_1 = [r_{11}, r_{12}, \dots, r_{1h} + 1, \dots, r_{1n_1}]$, $R'_s = [r_{s1}, r_{s2}, \dots, r_{st} - 1, \dots, r_{sn_s}]$ ($2 \leq s \leq k$) and $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$, ($2 \leq i \leq k$), $i \neq s$ are losing score lists of some k -partite hypertournament, t is the largest integer such that $r_{s(t-1)} < r_{st} = \dots = r_{sn_s}$.*

Proof. Let $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$ ($1 \leq i \leq k$) be losing score lists of a k -partite hypertournament M with vertex sets $U_i = \{u_{i1}, u_{i2}, \dots, u_{ij_i}\}$ so that $d^-(u_{ij_i}) = r_{ij_i}$ for each i ($1 \leq i \leq k$, $1 \leq j_i \leq n_i$).

Let h be the smallest integer such that

$$r_{11} = r_{12} = \dots = r_{1h} < r_{1(h+1)} \leq \dots \leq r_{1n_1}$$

and t be the largest integer such that

$$r_{s1} \leq r_{s2} \leq \dots \leq r_{s(t-1)} < r_{st} = \dots = r_{sn_s}$$

Now, let

$$R'_1 = [r_{11}, r_{12}, \dots, r_{1h} + 1, \dots, r_{1n_1}],$$

$$R'_s = [r_{s1}, r_{s2}, \dots, r_{st} - 1, \dots, r_{sn_s}]$$

($2 \leq s \leq k$), and $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$, ($2 \leq i \leq k$), $i \neq s$.

Clearly, R'_1 and R'_s are both in non-decreasing order.

Since $r_{1h} < \frac{\alpha_1}{n_1} \prod_1^k \binom{n_p}{\alpha_p}$, there is at least one $[\alpha_i]_1^k$ -arc e containing both u_{1h} and u_{st} with u_{st} as the last element in e , let $e' = (u_{1h}, u_{st})$. Clearly, R'_1 , R'_s

and $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$ for each i ($2 \leq i \leq k$), $i \neq s$ are the k losing score lists of $M' = (M - e) \cup e'$. \square

The next observation follows from Lemma 6, and the proof can be easily established.

Lemma 7 *Let $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$, ($1 \leq i \leq k$) be k non-decreasing sequences of non-negative integers satisfying (1). If $r_{1n_1} < \frac{\alpha_1}{n_1} \prod_1^k \binom{n_t}{\alpha_t}$, then there exists s and t ($2 \leq s \leq k$), $1 \leq t \leq n_s$ such that $R'_1 = [r_{11}, r_{12}, \dots, r_{1h} + 1, \dots, r_{1n_1}]$, $R'_s = [r_{s1}, r_{s2}, \dots, r_{st} - 1, \dots, r_{sn_s}]$ and $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$, ($2 \leq i \leq k$), $i \neq s$ satisfy (1).*

Proof of Theorem 3. Necessity. Let R_i , ($1 \leq i \leq k$) be the k losing score lists of a k -partite hypertournament $M(U_i, 1 \leq i \leq k)$. For any p_i with $\alpha_i \leq p_i \leq n_i$, let $U'_i = \{u_{ij_i}\}_{j_i=1}^{p_i}$ ($1 \leq i \leq k$) be the sets of vertices such that $d^-(u_{ij_i}) = r_{ij_i}$ for each $1 \leq j_i \leq p_i$, $1 \leq i \leq k$. Let M' be the k -partite hypertournament formed by U'_i for each i ($1 \leq i \leq k$).

Then,

$$\begin{aligned} \sum_{i=1}^k \sum_{j_i=1}^{p_i} r_{ij_i} &\geq \sum_{i=1}^k \sum_{j_i=1}^{p_i} d_{M'}^-(u_{ij_i}) \\ &= \prod_1^k \binom{p_t}{\alpha_t}. \end{aligned}$$

Sufficiency. We induct on n_1 , keeping n_2, \dots, n_k fixed. For $n_1 = \alpha_1$, the result is obviously true. So, let $n_1 > \alpha_1$, and similarly $n_2 > \alpha_2, \dots, n_k > \alpha_k$. Now,

$$\begin{aligned} r_{1n_1} &= \sum_{i=1}^k \sum_{j_i=1}^{n_i} r_{ij_i} - \left(\sum_{j_1=1}^{n_1-1} r_{1j_1} + \sum_{i=2}^k \sum_{j_i=1}^{n_i} r_{ij_i} \right) \\ &\leq \prod_1^k \binom{n_t}{\alpha_t} - \binom{n_1-1}{\alpha_1} \prod_2^k \binom{n_t}{\alpha_t} \\ &= \left[\binom{n_1}{\alpha_1} - \binom{n_1-1}{\alpha_1} \right] \prod_2^k \binom{n_t}{\alpha_t} \\ &= \binom{n_1-1}{\alpha_1-1} \prod_2^k \binom{n_t}{\alpha_t}. \end{aligned}$$

We consider the following two cases.

Case 1. $r_{1n_1} = \binom{n_1-1}{\alpha_1-1} \prod_2^k \binom{n_t}{\alpha_t}$. Then,

$$\begin{aligned} \sum_{j_1=1}^{n_1-1} r_{1j_1} + \sum_{i=2}^k \sum_{j_i=1}^{n_i} r_{ij_i} &= \sum_{i=1}^k \sum_{j_i=1}^{n_i} r_{ij_i} - r_{1n_1} \\ &= \prod_1^k \binom{n_t}{\alpha_t} - \binom{n_1-1}{\alpha_1-1} \prod_2^k \binom{n_t}{\alpha_t} \\ &= \left[\binom{n_1}{\alpha_1} - \binom{n_1-1}{\alpha_1-1} \right] \prod_2^k \binom{n_t}{\alpha_t} \\ &= \binom{n_1-1}{\alpha_1} \prod_2^k \binom{n_t}{\alpha_t}. \end{aligned}$$

By induction hypothesis $[r_{11}, r_{12}, \dots, r_{1(n_1-1)}]$, R_2, \dots, R_k are losing score lists of a k -partite hypertournament $M'(\mathcal{U}'_1, \mathcal{U}_2, \dots, \mathcal{U}_k)$ of order $\left(\sum_{i=1}^k n_i\right) - 1$.

1. Construct a k -partite hypertournament M of order $\sum_{i=1}^k n_i$ as follows. In M' , let $\mathcal{U}'_1 = \{u_{11}, u_{12}, \dots, u_{1(n_1-1)}\}$, $\mathcal{U}_i = \{u_{ij_i}\}_{j_i=1}^{n_i}$ for each i , ($2 \leq i \leq k$). Adding a new vertex u_{1n_1} to \mathcal{U}'_1 , for each $\left(\sum_{i=1}^k \alpha_i\right)$ -tuple containing u_{1n_1} , arrange u_{1n_1} on the last entry. Denote E_1 to be the set of all these $\binom{n_1-1}{\alpha_1-1} \prod_2^k \binom{n_t}{\alpha_t} \left(\sum_{i=1}^k \alpha_i\right)$ -tuples. Let $E(M) = E(M') \cup E_1$. Clearly, R_i for each i , ($1 \leq i \leq k$) are the k losing score lists of M .

Case 2. $r_{1n_1} < \binom{n_1-1}{\alpha_1-1} \prod_2^k \binom{n_t}{\alpha_t}$.

Applying Lemma 7 repeatedly on R_1 and keeping each R_i , ($2 \leq i \leq k$) fixed until we get a new non-decreasing list $R'_1 = [r'_{11}, r'_{12}, \dots, r'_{1n_1}]$ in which now $r'_{1n_1} = \binom{n_1-1}{\alpha_1-1} \prod_2^k \binom{n_t}{\alpha_t}$. By Case 1, R'_1, R_i ($2 \leq i \leq k$) are the losing score lists of a k -partite hypertournament. Now, apply Lemma 6 on R'_1, R_i ($2 \leq i \leq k$) repeatedly until we obtain the initial non-decreasing lists R_i for each i ($1 \leq i \leq k$). Then by Lemma 6, R_i for each i ($1 \leq i \leq k$) are the losing score lists of a k -partite hypertournament. \square

Proof of Theorem 4. Let $S_i = [s_{ij_i}]_{j_i=1}^{n_i}$ ($1 \leq i \leq k$) be the k score lists of a k -partite hypertournament $M(\mathcal{U}_i, 1 \leq i \leq k)$, where $\mathcal{U}_i = \{u_{ij_i}\}_{j_i=1}^{n_i}$ with

$d_M^+(u_{ij_i}) = s_{ij_i}$, for each i , ($1 \leq i \leq k$). Clearly,

$$d^+(u_{ij_i}) + d^-(u_{ij_i}) = \frac{\alpha_i}{n_i} \prod_1^k \binom{n_t}{\alpha_t}, (1 \leq i \leq k, 1 \leq j_i \leq n_i).$$

Let $r_{i(n_i+1-j_i)} = d^-(u_{ij_i})$, ($1 \leq i \leq k, 1 \leq j_i \leq n_i$).

Then $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$ ($i = 1, 2, \dots, k$) are the k losing score lists of M . Conversely, if R_i for each i ($1 \leq i \leq k$) are the losing score lists of M , then S_i for each i , ($1 \leq i \leq k$) are the score lists of M . Thus, it is enough to show that conditions (1) and (2) are equivalent provided $s_{ij_i} + r_{i(n_i+1-j_i)} = \left(\frac{\alpha_i}{n_i}\right) \prod_1^k \binom{n_t}{\alpha_t}$, for each i ($1 \leq i \leq k$ and $1 \leq j_i \leq n_i$).

First assume (2) holds. Then,

$$\begin{aligned} \sum_{i=1}^k \sum_{j_i=1}^{p_i} r_{ij_i} &= \sum_{i=1}^k \sum_{j_i=1}^{p_i} \left(\frac{\alpha_i}{n_i}\right) \left(\prod_1^k \binom{n_t}{\alpha_t}\right) - \sum_{i=1}^k \sum_{j_i=1}^{p_i} s_{i(n_i+1-j_i)} \\ &= \sum_{i=1}^k \sum_{j_i=1}^{p_i} \left(\frac{\alpha_i}{n_i}\right) \left(\prod_1^k \binom{n_t}{\alpha_t}\right) - \left[\sum_{i=1}^k \sum_{j_i=1}^{n_i} r_{ij_i} - \sum_{i=1}^k \sum_{j_i=1}^{n_i-p_i} s_{ij_i} \right] \\ &\geq \left[\sum_{i=1}^k \sum_{j_i=1}^{p_i} \left(\frac{\alpha_i}{n_i}\right) \left(\prod_1^k \binom{n_t}{\alpha_t}\right) \right] \\ &\quad - \left[\left(\left(\sum_1^k \alpha_i \right) - 1 \right) \prod_1^k \binom{n_i}{\alpha_i} \right] \\ &\quad + \sum_{i=1}^k (n_i - p_i) \left(\frac{\alpha_i}{n_i}\right) \prod_1^k \binom{n_t}{\alpha_t} \\ &\quad + \prod_1^k \binom{n_i - (n_i - p_i)}{\alpha_i} - \prod_1^k \binom{n_i}{\alpha_i} \\ &= \prod_1^k \binom{n_i}{\alpha_i}, \end{aligned}$$

with equality when $p_i = n_i$ for each i ($1 \leq i \leq k$). Thus (1) holds.

Now, when (1) holds, using a similar argument as above, we can show that (2) holds. This completes the proof. \square

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